MODULI STABILIZATION FROM FLUXES

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Abstract Compactifications of IIB string theory with internal NS and RR three-form flux are computationally attractive in that the lifting of moduli is via a *perturbative* and often explicitly calculable (super)potential. We focus on the T^6/Z_2 orientifold, and provide an illustative $\mathcal{N} = 2$ example. For other choices of flux, the resulting equations of motion can be solved to yield $\mathcal{N} = 0, 1, 2$ or 3 supersymmetry in four dimensions. (arXiv: 0810.5197, CALT-68-2441)

Moduli are a familiar by-product of string compactifications, but they do not exist in nature. Massless or nearly massless gravitationally coupled scalars generate long range interactions that have been excluded by fifth force experiments [1]. They are also problematic in cosmology [2].[1]

In traditional $\mathcal{N} = 1$ heterotic compactifications, perturbatively massless moduli can be lifted by well-known nonperturbative effects like world-sheet instantons or Euclidean NS five-brane instantons. However, the computational difficulty of quantitatively understanding these effects has so far proven insurmountable. There does not seem to exist a single compact example in which anyone has computed the relevant instanton sums and explicitly found a supersymmetric minimum of the resulting potential.

In contrast, when one turns on NS and RR three-form flux through nontrivial three-cycles in a four-dimensional compactification of IIB string theory, many of the moduli are lifted by a *perturbative* scalar potential [4]

$$\mathcal{V} \propto \int d^6y \sqrt{g_6}\, |G^{\text{ISD}}|^2, \quad G = F^{\text{RR}}_{(3)} - \varphi H^{\text{NS}}_{(3)}. \tag{1.1}$$

[1] As the early universe cools, approximate moduli can easily overshoot the minima of their potentials. When this happens, they contribute an energy density like that of matter rather than radiation, causing deviations from the successful predictions of big bang nucleosynthesis.

Here, φ is the dilaton-axion and $G^{\rm ISD}$ is the imaginary-anti-self-dual part of the complex flux G.[2] This potential descends from the compact part of the flux kinetic terms in the ten-dimensional IIB supergravity action.

For consistency of the compactification, the fluxes must satisfy a D3-brane charge tadpole cancellation condition,

$$N_{\rm D3} + \frac{1}{2(2\pi)^4\alpha'^2}\int H^{\rm NS}_{(3)} \wedge F^{\rm RR}_{(3)} - \frac{1}{4}N_{\rm O3} = 0. \qquad (1.2)$$

in which wedged fluxes contribute in exactly the same way as space-filling D3-branes and O3-planes.[3] An obvious way to satisfy this condition is to start with a consistent string compactification involving space-filling D3-branes, and then construct new vacua by simply trading off D3-branes for fluxes.

However, whereas space-filling D3-branes and O3-planes preserve $\mathcal{N}=4$ supersymmetry in four dimensions, the fluxes preserve less. To preserve at least $\mathcal{N}=1$ supersymmetry, we need vanishing dilatino variation, and vanishing gravitino variation for at least one gravitino. This implies that G must be primitive (*i.e.*, $J\wedge G=0$, with J the Kähler form), and of type (2,1).[4]

The (2,1) condition can be imposed by a superpotential [5]

$$W = \int G \wedge \Omega, \qquad (1.3)$$

where Ω is the holomorphic (3,0) form. For proper Calabi-Yau compactification, the primitivity condition is trivial due to the absence of a fifth cohomology class. More generally, it is a linear constraint on the Kähler moduli which is easy to solve. On the other hand, the (2,1) condition is not so simple. When the periods of the holomorphic (3,0) form are known, we can compute the superpotential (1.3). But for Calabi-Yau orientifolds or F-theory compactifications, this generally involves complicated trancendental functions, for which it has not been possible to vary the superpotential and solve the resulting equations, except near singular (conifold) points in the moduli space of complex structure.

Still, one might expect that the equations of motion *are* soluble when we choose a simple enough compactification manifold, and simplest choice is a torus. That is the choice we will make here, with one modification. Since we would like to turn on flux, Eq. (1.2) requires that there also be orientifold

[2] Since the compact manifold is six-dimensional, the hodge star operator squares to -1 and its eigenvalues are $\pm i$. The corresponding eigenfunctions are *imaginary-self-dual* (ISD) three-forms ($*a_{(3)} = ia_{(3)}$) and *imaginary-anti-self-dual* (IASD) three-forms ($*a_{(3)} = -ia_{(3)}$).

[3] In F-theory compactifications, there would also be a contribution to Eq. (1.2) from the Euler character of the fourfold, and from instantons on the compact part of D7-brane worldvolumes.

[4] It can be shown that if G is (2,1) and primitive then it is also ISD. So, if the supersymmetry conditions are satisfied, then the scalar potential (1.1) is automaticaly minimized and equal to zero.

planes. So, the compactification that we will consider is on the torus orientifold T^6/Z_2 [3, 6].

To define the T^6/Z_2 orientifold, we first compactify on T^6, defined by $x^i \cong x^i + 1$, $y^i \cong y^i + 1$, $i = 1, 2, 3$. Then, we mod out by the Z_2 parity operation $\Omega R_6 (-1)^{F_L}$. Here, Ω is worldsheet parity, R_6 is a reflection of all of the T^6 coordinates, and $(-1)^{F_L}$ is a parity operation that is required by supersymmetry.[5] The massless states that survive the orientifold projection are the four-dimensional graviton $g_{\mu\nu}$, the scalars g_{ab}, C_{abcd} and φ, and the twelve $U(1)$ gauge bosons $B_{a\mu}$ and $C_{a\mu}$. (Here C denotes a RR potential, and B the NS potential). It is also consistent with the orientifold projection to turn on internal NS and RR three-form fluxes. However, note that these fluxes are discrete by Dirac quantization, and non-dynamical in the massless sector, since the corresponding zero-modes are projected out.

In the absence of flux, this orientifold describes the same theory as Type I on T^6, via T-duality in all six torus directions. The sixteen D9-branes of $SO(32)$ in Type I become sixteen D3-branes after T-duality. Also, the charge and tension of the D3-branes is cancelled by 2^6 O3-planes located at the fixed points of the Z_2. So, the low energy effective field theory is the same $\mathcal{N} = 4$ $SO(32)$ super-Yang-Mills, coupled to $\mathcal{N} = 4$ supergravity.

Once we replace some of the D3-branes with fluxes, this story is modified. The fluxes generate a potential for the scalars, and correspond to turning on charges that couple the scalars to the twelve $U(1)$ gauge fields. The result is a superhiggs mechanism in which many of the scalars get massive or are eaten by massive vectors, breaking $\mathcal{N} = 4$ to $\mathcal{N} < 4$ supersymmetry.

As an example, consider the choice of flux [6]

$$\frac{1}{(2\pi)^2\alpha'} F^{\mathrm{RR}}_{(3)} = 4dx^1 \wedge dx^2 \wedge dy^3 + 4dy^1 \wedge dy^2 \wedge dy^3, \tag{1.4}$$

$$\frac{1}{(2\pi)^2\alpha'} H^{\mathrm{NS}}_{(3)} = 4dx^1 \wedge dx^2 \wedge dx^3 + 4dy^1 \wedge dy^2 \wedge dx^3. \tag{1.5}$$

Let us parametrize the complex structure as $dz^i = dx^i + \tau^i{}_j dy^j$, and normalize the holomorphic three-form so that $\Omega = dz^1 \wedge dz^2 \wedge dz^3$. Then, by wedging the appropriate three-forms together, it is easy to show that

$$W = \int G \wedge \Omega \, \propto \, 1 + (\operatorname{cof}\tau)_3{}^3 + \varphi\Big(\det\tau + \tau^3{}_3\Big). \tag{1.6}$$

For supersymmetric vacua, the equations of motion are that $D_I W = \partial_I W + (\partial_I \mathcal{K}) W = 0$, where $\mathcal{K}(\varphi^I, \bar{\varphi}^{\bar{I}})$ is the Kähler potential on moduli space. Since

[5] For massless modes, $(-1)^{F_L}$ acts as -1 on left-moving Ramond sector states and $+1$ otherwise. If this factor were not included, the resulting spectrum of states would not fill out supergravity multiplets.

the superpotential is independent of Kähler moduli, this simplifies to $W = \partial_\varphi W = \partial_\tau W = 0$, which through a small amount of algebra can be shown to imply that

$$\varphi\,\tau^3{}_3 = -1, \quad \tau^1{}_1\tau^2{}_2 - \tau^1{}_2\tau^2{}_1 = -1. \tag{1.7}$$

So, the moduli space of complex structure is complex four-dimensional, and can be parametrized by, say $\tau^1{}_1$, $\tau^2{}_2$, $\tau^3{}_3$, and $\tau^1{}_2$. In Eq. (1.4), we expressed the flux as a linear combination of integral three-forms with integer coefficients, as required Dirac quantization. Using Eq. (1.7), we can also write the flux in terms of holomorphic and antiholomorphic forms. Restricting to $\tau^i{}_j$ diagonal and imaginary for simplicity, we find that

$$G \propto dz^1 \wedge d\bar{z}^{\bar{2}} \wedge dz^3 + d\bar{z}^{\bar{1}} \wedge dz^2 \wedge dz^3. \tag{1.8}$$

This makes it clear that if Eq. (1.7) is satisfied then the complex flux is indeed of type (2,1). In addition, it is easy to show that the primitivity condition is satisfied on the appropriate subspace of Kähler moduli.

As a final remark, note that if we replace z^1 and z^2 by their complex conjugates, then the flux (1.8) is still (2,1) and primitive. In other words, there are two inequivalent complex structures in which the conditions for $\mathcal{N} = 1$ supersymmetry are satisfied. This implies that the solution is actually $\mathcal{N} = 2$ supersymmetric, and shows how to engineer solutions with anywhere from $\mathcal{N} = 0$ supersymmetry (when there is *no* solution to $D_I W = 0$) to $\mathcal{N} = 3$ supersymmetry (when the solution permits three independent complex structures).

For a more complete discussion, including large classes of $\mathcal{N} = 1$ solutions, we refer the reader to the work [6] on which this review is based, and the references contained therein.

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